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Quantal symmetry for a system with a singular higher-order Lagrangian

Zi-ping Li†‡ and Zheng-wen Long‡

† CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China
‡ Department of Applied Physics, Beijing Polytechnic University, Beijing 100022, People's Republic of China

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Abstract. Based on the phase-space generating functional of the Green function for a constrained Hamiltonian system with a singular higher-order Lagrangian, the canonical Ward identities for such a system under the local and global transformation have been derived, respectively. The quantal conserved charge (QCC) under the global symmetry transformation is also deduced. In general, these QCCs are different from the Noether charge in classical theory. A comparison of these quantal conservation laws with those deriving from the configuration-space path integral for gauge-invariant theories is discussed. We give a preliminary application of our results to Yang–Mills (YM) theory and Chern–Simons (CS) theory with higher-order derivatives. A new form of gauge-ghost proper vertices and new conserved charges at the quantum level are obtained for the YM theory; the quantal Becchi–Rouet–Stora (BRS) conserved charges and conserved angular momentum are also derived for CS theory. The advantage of our canonical formalism is that we do not carry out the integration over the canonical momenta in the phase-space path integral as usually performed.

1. Introduction

Symmetry is now a fundamental concept in modern physics. The connection between continuous global symmetries and conservation laws are usually referred to as the first Noether theorem in classical theory. The classical second Noether theorem or Noether identity refers to invariance of the action integral of the system under a local transformation parametrized by r arbitrary functions and their derivatives. If an action is invariant under such transformation, then there are r differential identities (Noether identities) which involve the functional derivatives of the action integral. In quantum theory, the Noether identity corresponds to the Ward (or Ward–Takahashi) identity. Noether theorems and their generalization are usually formulated in terms of Lagrangian variables in configuration space [1, 2]. The identities relating to the Green function in QED were obtained by Ward [3] and Takahashi [4]. In non-Abelian theories, their role is played by the so-called generalized Ward identities, first obtained by Slavonov [5] and Taylor [6]. Ward identities and their generalization play an important role in modern field theories. They are useful tools for the renormalization of field theories and calculation in practical problems (for example, in QCD). Ward identities have been generalized to the supersymmetry [7] and superstring theories [8] and other problems. All these derivations for Ward identities in the functional integration method are usually performed by using a configuration-space generating functional [9–11] which is valid for the case when the integration in the phase-space path integral over the canonical momenta belongs to the Gauss-type category. Phase-space path integrals are more basic than configuration-space

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path integrals: the latter provide for a Hamiltonian quadratic in the canonical momenta, whereas the former apply to arbitrary Hamiltonians. Thus, the phase-space form of the path integral is a necessary precursor to the configuration space form of the path integral [12]. For certain cases where the phase-space path integral can be simplified by carrying out explicit integration over canonical momenta in which the 'mass' depends on coordinates [13, 14] or depends on coordinates and momenta [15], the effective Lagrangians in configuration space are singularities with a delta function. For the constrained Hamiltonian system with complicated constraints, especially for the system with singular high-order Lagrangian, it is very difficult or even impossible to carry out the integration over canonical momenta. The dynamical systems described in terms of high-order Lagrangians obtained by many authors are of much interest in connection with gauge theories, gravity, supersymmetry, string models and other problems [16]. Therefore, investigation of the symmetry properties of the system in the phase space has more basic sense. Based on the invariance of the phase-space generating functional of the Green function for a system with a singular first-order Lagrangian under the local transformation of canonical variables in extended phase space, the canonical Ward identity (CWI) for such a system has been studied by one of the authors in a previous work [17]; for a system with a singular higher-order Lagrangian a brief discussion has also been given [18]. The global quantal canonical symmetry for a system with a singular first-order Lagrangian had also been considered [19]. Here, the local and global quantal symmetry at the quantum level for a system with a singular high-order Lagrangian will be studied further.

The paper is organized as follows. In section 2, based on the phase-space generating functional of the Green function for a system with a singular higher-order Lagrangian, the CWIs have been derived under the local and global transformation of canonical variables, respectively. In section 3, the quantal conserved charge (QCC) under the global symmetry transformation is also deduced. In general, these QCCs differ from classical Noether charges. The connection between the symmetries and conservation laws in classical theories in general is no longer preserved in quantum theories. In section 4, for the gauge-invariant system, the quantal conservation laws connecting with the global symmetry in configuration-space path integral deriving from the Faddeev–Popov (FP) trick is studied. In section 5, we give a preliminary application of our results to Yang–Mills (YM) theory with higher-order derivatives; a new form of gauge-ghost proper vertices are derived from both the CWIs for local and global transformation, respectively. These gauge-ghost proper vertices differ from the usual Ward identity arising from BRS invariance. Some new QCCs have also been obtained for higher-derivative YM theory. The application of our formulation to non-Abelian higher-derivative CS theory is discussed in section 6, and the quantal BRS conserved charge and conserved angular momenta are derived. The results arising from the configuration-space generating functional coincide with the results deriving from the phase-space generating functional. This conserved angular momentum at the quantum level differs from the classical Noether one in that one needs to take into account the contribution of the angular momenta of the ghost fields. The problem of fraction spin in non-Abelian CS theory needs further study.

2. CWIs

2.1. Preliminaries

In order to formulate the path integral quantization for a system with a singular higher-order Lagrangian, we start this section by reviewing very briefly the transformation from Lagrangian to Hamiltonian formalism for such a system [16].

Let us consider a physical field defined by the field function $\varphi^\alpha(x)(\alpha = 1, 2, \ldots, n)$,

$x = (x_0, x_i)$, $(x_0 = t, i = 1, 2, 3)$, where the motion of the field is described by a Lagrangian involving high-order derivatives in the form of a functional

$$L = L[\varphi^\alpha_{(0)}, \varphi^\alpha_{(1)}, \ldots, \varphi^\alpha_{(N)}] = \int \mathrm{d}^3x\, \mathcal{L}(\varphi^\alpha, \varphi^\alpha_{,\mu}, \ldots, \varphi^\alpha_{,\mu(N)}) \tag{1}$$

where

$$\varphi^\alpha_{(0)} = \varphi^\alpha \qquad \varphi^\alpha_{(1)} = \dot{\varphi} \qquad \varphi^\alpha_{(2)} = \ddot{\varphi}, \ldots, \varphi^\alpha_{,\mu} = \frac{\partial}{\partial x^\mu}\varphi^\alpha \qquad \varphi^\alpha_{,\mu(m)} = \underbrace{\partial_\mu, \ldots, \partial_\sigma}_{m} \varphi^\alpha.$$

Using the Ostrogradsky transformation, we can transform the Lagrangian formalism to Hamiltonian formalism for a system with a singular higher-order Lagrangian [16]. One introduces the generalized canonical momenta

$$\pi_{\alpha/m} = \sum_{j=0}^{N-m} (-1)^j \frac{\mathrm{d}^j}{\mathrm{d}t^j} \frac{\delta L}{\delta \varphi^\alpha_{(j+m)}} \tag{2}$$

and using these relations one can move from the Lagrangian description to the Hamiltonian description. The canonical Hamiltonian is defined by

$$H_c[\varphi^\alpha_{(m-1)}, \pi_{\alpha/m}] = \int \mathrm{d}^3x \mathcal{H}_c = \int \mathrm{d}^3x (\pi_{\alpha/m}\varphi^\alpha_{(m)} - \mathcal{L}) \tag{3}$$

which may be formed by eliminating only the highest derivatives $\varphi^\alpha_{(N)}$. The summation over indices α from 1 to n, m from 1 to N is taken repeatedly. For the singular Lagrangian L, the extended Hessian matrix $(H_{\alpha\beta})$ is degenerate:

$$\det |H_{\alpha\beta}| = \det |\delta^2 L/\delta\varphi^\alpha_{(N)}\delta\varphi^\beta_{(N)}| = 0$$

hence one cannot solve all $\varphi^\alpha_{(N)}$ from the definition of the canonical momenta. Then there are constraints among the canonical variables in phase space [20]:

$$\Phi^0_\alpha(\varphi^\alpha_{(m-1)}, \pi_{\alpha/m}) \approx 0 \qquad (\alpha = 1, 2, \ldots, n - R) \tag{4}$$

where the sign $\approx$ (weak equality) means equality on the constrained hypersurface, the rank of extended Hessian matrix is assumed to be R. Equation (4) represents the primary constraints. Thus, a system with a singular higher-order Lagrangian is subject to some inherent phase-space constraints and is called a generalized constrained Hamiltonian system. From the stationary of constraint, one can define successively the secondary constraints from the primary ones. The process of the consistency requirements will terminate at some stage when new constraints no longer appear. All the constraints are classified into two classes. The constraints in the first class are those whose generalized Poisson bracket with any of the constraints are equal to zero or equal to the linear combination of the constraints; if this is not the case, the constraint is called second class.

Let $\Lambda_k(\varphi^\alpha_{(m-1)}, \pi_{\alpha/m}) \approx 0$ $(k = 1, 2, \ldots, K)$ be first-class constraints, and $\theta_i(\varphi^\alpha_{(m-1)}, \pi_{\alpha/m}) \approx 0$ $(i = 1, 2, \ldots, I)$ be second-class constraints. According to the rule of path integral quantization, for each first-class constraint, one must choose a gauge condition. The phase-space generating functional of the Green function for a system with a singular higher-order Lagrangian can be written as [21]

$$Z[j, K] = \int \mathcal{D}\varphi^\alpha_{(m-1)}\mathcal{D}\pi_{\alpha/m}\delta(\Phi)\sqrt{\det\{\Phi, \Phi\}} \times \exp\left\{\mathrm{i}\int \mathrm{d}^4x[\pi_{\alpha/m}\varphi^\alpha_{(m)} - \mathcal{H}_c + j^{m-1}_\alpha\varphi^\alpha_{(m-1)} + K^\alpha_m\pi_{\alpha/m}]\right\} \tag{5}$$

where $\mathcal{H}_c$ is a canonical Hamiltonian density, $\Phi = \{\Phi_n\}$ is a set of all constraints (for a theory with second-class constraints) or the set of constraints and gauge conditions (for a theory with first-class constraints), j_α^{m-1} and K_m^α are exterior sources with respect to $\varphi^\alpha_{(m-1)}$ and $\pi_{\alpha/m}$, respectively. Using the properties of the δ-function and the integral over Grassmann variables $C_l(x)$ and $\bar{C}_k(x)$, one gets

$$Z[j,K] = \int \mathcal{D}\varphi^\alpha_{(m-1)}\mathcal{D}\pi_{\alpha/m}\mathcal{D}\lambda_m\mathcal{D}\bar{C}_k\mathcal{D}C_l \times \exp\left\{\mathrm{i}\int \mathrm{d}^4x[\mathcal{L}^P_{eff} + j_\alpha^{m-1}\varphi^\alpha_{(m-1)} + K_m^\alpha\pi_{\alpha/m}]\right\} \tag{6}$$

where

$$\mathcal{L}^P_{eff} = \pi_{\alpha/m}\varphi^\alpha_{(m)} - \mathcal{H}_c + \lambda_n\Phi_n + \tfrac{1}{2}\int \mathrm{d}^4x\bar{C}_k(x)\{\Phi_k(x),\Phi_l(y)\}C_l(y) \tag{7}$$

and $\lambda_n(x)$ are Lagrange multipliers.

2.2. CWI for local transformation

For the sake of simplicity, we put $\phi^\alpha_{(m-1)} = (\phi^\alpha_{(m-1)}, \lambda_n, \bar{C}_k, C_l)$, $J_\alpha^{m-1} = (j_\alpha^{m-1}, \eta_n, \xi_k, \bar{\xi}_l)$ where η_n, ξ_k and $\bar{\xi}_l$ are exterior sources with respect to λ_n, $\bar{C}_k$ and C_l, respectively; then expression (6) can be written as

$$Z[J,K] = \int \mathcal{D}\phi^\alpha_{(m-1)}\mathcal{D}\pi_{\alpha/m}\exp\left\{\mathrm{i}\int \mathrm{d}^4x[\mathcal{L}^P_{eff} + J_\alpha^{m-1}\phi^\alpha_{(m-1)} + K_m^\alpha\pi_{\alpha/m}]\right\}. \tag{8}$$

Let us consider an infinitesimal transformation in extended phase space

$$\begin{aligned}
&x^\mu = x^\mu + \Delta x^\mu = x^\mu + R^\mu_\sigma\varepsilon^\sigma(x)\\
&\phi'^\alpha_{(m-1)}(x') = \phi^\alpha_{(m-1)}(x) + \Delta\phi^\alpha_{(m-1)}(x) = \phi^\alpha_{(m-1)}(x) + S^\alpha_{\sigma(m-1)}\varepsilon^\sigma(x)\\
&\pi'_{\alpha/m}(x') = \pi_{\alpha/m}(x) + \Delta\pi_{\alpha/m}(x) = \pi_{\alpha/m}(x) + T_{\sigma\alpha/m}\varepsilon^\sigma(x)
\end{aligned} \tag{9}$$

where $\varepsilon^\sigma(x)(\sigma = 1,2,\ldots,r)$ are infinitesimal arbitrary functions, whose values and their derivatives up to required order will vanish on the boundary of the time–space domain. R^α_σ, $S^\alpha_{\sigma(m-1)}$ and $T_{\sigma\alpha/m}$ are linear differential operators

$$\begin{aligned}
&R^\mu_\sigma = a^{\mu\nu(k)}_\sigma\partial_{\nu(k)} \qquad S^\alpha_{\sigma(m-1)} = b^{\alpha\nu(l)}_{\sigma(m-1)}\partial_{\nu(l)} \qquad T_{\sigma\alpha/m} = c^{\nu(m)}_{\sigma\alpha/m}\partial_{\nu(m)}\\
&\nu(n) = \underbrace{\mu\nu,\ldots,\rho\sigma}_{n} \qquad \partial_{\nu(n)} = \underbrace{\partial_\mu\partial_\nu,\ldots,\partial_\rho\partial_\sigma}_{n}.
\end{aligned} \tag{10}$$

a, b and c are functions of x, $\phi^\alpha_{(m-1)}$ and $\pi_{\alpha/m}$. The variation of the effective canonical action $I^P_{eff} = \int \mathrm{d}^4x\mathcal{L}^P_{eff}$ under transformation (9) is given by [16]

$$\begin{aligned}
\delta I^P_{eff} = \int \mathrm{d}^4x\bigg\{&\frac{\delta I^P_{eff}}{\delta\phi^\alpha_{(m-1)}}\delta\phi^\alpha_{(m-1)} + \frac{\delta I^P_{eff}}{\delta\pi_{\alpha/m}}\delta\pi_{\alpha/m} + \partial_\mu[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_c)\Delta x^\mu]\\
&+D[\pi_{\alpha/m}\delta\phi^\alpha_{(m-1)}]\bigg\}
\end{aligned} \tag{11}$$

where $D = \mathrm{d}/\mathrm{d}t$, and

$$\delta\phi^\alpha_{(m-1)} = \Delta\phi^\alpha_{(m-1)} - \phi^\alpha_{(m-1),\mu}\Delta x^\mu \qquad \delta\pi_{\alpha/m} = \Delta\pi_{\alpha/m} - \pi_{\alpha/m,\mu}\Delta x^\mu \tag{12}$$

$$\frac{\delta I^P_{eff}}{\delta\phi^\alpha_{(m-1)}} = -\dot{\pi}_{\alpha/m} - \frac{\delta H^P_{eff}}{\delta\phi^\alpha_{(m-1)}} \qquad \frac{\delta I^P_{eff}}{\delta\pi_{\alpha/m}} = \dot{\phi}^\alpha_{(m-1)} - \frac{\delta H^P_{eff}}{\delta\pi_{\alpha/m}} \tag{13}$$

and H^P_{eff} is a Hamiltonian connected with $L^P_{eff} = \int \mathrm{d}^4x\mathcal{L}^P_{eff}$. The Jacobian of transformation (9) is denoted by $\bar{J}[\phi,\pi,\varepsilon]$. For the invariant of the generating functional (8) under transformation (9), one has

$$Z[J, K] = \int \mathcal{D}\phi^{\alpha}_{(m-1)}\mathcal{D}\pi_{\alpha/m}\bar{J}[\phi, \pi, \varepsilon]\left\{1 + \mathrm{i}\delta I^{P}_{eff} + \mathrm{i}\int \mathrm{d}^4x(J^{m-1}_{\alpha}\delta\phi^{\alpha}_{(m-1)} + K^{\alpha}_{m}\delta\pi_{\alpha/m}) \right.$$
$$\left. +\partial_{\mu}[(J^{m-1}_{\alpha}\phi^{\alpha}_{(m-1)} + K^{\alpha}_{m}\pi_{\alpha/m})\Delta x^{\mu}]\right\}$$
$$\times \exp\left\{\mathrm{i}\int \mathrm{d}^4x[\mathcal{L}^{P}_{eff} + J^{m-1}_{\alpha}\phi^{\alpha}_{(m-1)} + K^{\alpha}_{m}\pi_{\alpha/m}]\right\}. \tag{14}$$

This invariance implies that

$$\left.\frac{\delta Z[J, K]}{\delta\varepsilon_{\sigma}}\right|_{\varepsilon_{\sigma}=0} = 0.$$

Using the boundary conditions of $\varepsilon_{\sigma}(x)$ and functionally differentiating (14) with respect to $\varepsilon_{\sigma}(x)$ and setting $J^{m-1}_{\alpha} = K^{\alpha}_{m} = 0$, one obtains

$$\int \mathcal{D}\phi^{\alpha}_{(m-1)}\mathcal{D}\pi_{\alpha/m}\left[J^{0}_{\sigma} + \tilde{S}^{\alpha}_{\sigma(m-1)}\left(\frac{\delta I^{P}_{eff}}{\delta\phi^{\alpha}_{(m-1)}}\right) - \tilde{R}^{\mu}_{\sigma}\left(\phi^{\alpha}_{(m-1),\mu}\frac{\delta I^{P}_{eff}}{\delta\phi^{\alpha}_{(m-1)}}\right)\right.$$
$$\left. + \tilde{T}_{\sigma\alpha/m}\left(\frac{\delta I^{P}_{eff}}{\delta\pi_{\alpha/m}}\right) - \tilde{R}^{\mu}_{\sigma}\left(\pi_{\alpha/m,\mu}\frac{\delta I^{P}_{eff}}{\delta\pi_{\alpha/m}}\right)\right]\exp(\mathrm{i}I^{P}_{eff}) = 0 \tag{15}$$

where

$$J^{0}_{\sigma} = -\mathrm{i}\delta\bar{J}[\phi, \pi, \varepsilon]/\delta\varepsilon_{\sigma}|_{\varepsilon_{\sigma}=0}$$

and $\tilde{R}^{\mu}_{\sigma}$, $\tilde{S}^{\alpha}_{\sigma(m-1)}$ and $\tilde{T}_{\sigma\alpha/m}$ are adjoint operators with respect to R^{μ}_{σ}, $S^{\alpha}_{\sigma(m-1)}$ and $T_{\sigma\alpha/m}$, respectively [22]. Using the boundary conditions of $\varepsilon_{\sigma}(x)$ and differentiating (14) with respect to $\varepsilon_{\sigma}(x)$ one can also obtain the CWI for a system with a singular higher-order Lagrangian for the case $J^{0}_{\sigma} = 0$:

$$\left[\tilde{S}^{\alpha}_{\sigma(m-1)}\left(\frac{\delta I^{P}_{eff}}{\delta\phi^{\alpha}_{(m-1)}}\right) - \tilde{R}^{\mu}_{\sigma}\left(\phi^{\alpha}_{(m-1),\mu}\frac{\delta I^{P}_{eff}}{\delta\phi^{\alpha}_{(m-1)}}\right) + \tilde{S}^{\alpha}_{\sigma(m-1)}J^{m-1}_{\alpha} - \tilde{R}^{\mu}_{\sigma}(\phi^{\alpha}_{(m-1),\mu}J^{m-1}_{\alpha})\right.$$
$$+\tilde{T}_{\sigma\alpha/m}\left(\frac{\delta I^{P}_{eff}}{\delta\pi_{\alpha/m}}\right) - \tilde{R}^{\mu}_{\sigma}\left(\pi_{\alpha/m,\mu}\frac{\delta I^{P}_{eff}}{\delta\pi_{\alpha/m}}\right) + \tilde{T}_{\sigma\alpha/m}K^{\alpha}_{m}$$
$$\left.-\tilde{R}^{\mu}_{\sigma}(\pi_{\alpha/m,\mu}K^{\alpha}_{m})\right]_{\substack{\phi^{\alpha}_{(m-1)}\to\frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}} \\ \pi_{\alpha/m}\to\frac{\delta}{\mathrm{i}\delta K^{\alpha}_{m}}}} \times Z[J, K] = 0. \tag{16}$$

Functionally differentiating (16) with respect to the exterior sources J^{0}_{α} many times and setting all exterior sources equal to zero, one can obtain some relationships among the Green functions.

2.3. *CWI for global transformation*

Let us consider a global symmetry transformation in extended phase space whose infinitesimal transformation is given by

$$\begin{aligned}
&x^{\mu'} = x^{\mu} + \Delta x^{\mu} = x^{\mu} + \varepsilon_{\sigma}\tau^{\mu\sigma}(x, \phi^{\alpha}_{(m-1)}, \pi_{\alpha/m}) \\
&\phi^{\alpha'}_{(m-1)}(x') = \phi^{\alpha}_{(m-1)}(x) + \Delta\phi^{\alpha}_{(m-1)}(x) = \phi^{\alpha}_{(m-1)}(x) + \varepsilon_{\sigma}\xi^{\mu\sigma}_{(m-1)}(x, \phi^{\alpha}_{(m-1)}, \pi_{\alpha/m}) \\
&\pi'_{\alpha/m}(x') = \pi_{\alpha/m}(x) + \Delta\pi_{\alpha/m}(x) = \pi_{\alpha/m}(x) + \varepsilon_{\sigma}\eta^{\sigma}_{\alpha/m}(x, \phi^{\alpha}_{(m-1)}, \pi_{\alpha/m})
\end{aligned} \tag{17}$$

where $\varepsilon_{\sigma}(\sigma = 1, 2, \ldots, r)$ are infinitesimal arbitrary parameters, $\tau^{\mu\sigma}$, $\xi^{\alpha\sigma}_{(m-1)}$ and $\eta^{\sigma}_{\alpha/m}$ are some functions of x, $\phi^{\alpha}_{(m-1)}$ and $\pi_{\alpha/m}$. It is supposed that the effective canonical action is invariant under transformation (17) and that the Jacobian of transformation (17) is equal to unity. According to the generating functional (8) is invariant under transformation (17); thus, we have

$$
\begin{aligned}
Z[J, K] = \int \mathcal{D}\phi^{\alpha}_{(m-1)}\mathcal{D}\pi_{\alpha/m}\Bigg(1 + \mathrm{i}\varepsilon_{\sigma}\int \mathrm{d}^4\Big\{ & J^{m-1}_{\alpha}(\xi^{\alpha\sigma}_{(m-1)} - \phi^{\alpha}_{(m-1),\mu}\tau^{\mu\sigma}) \\
& + K^{\alpha}_{m}(\eta^{\sigma}_{\alpha/m} - \pi_{\alpha/m,\mu}\tau^{\mu\sigma}) + \partial_{\mu}[(J^{m-1}_{\alpha}\phi^{\alpha}_{(m-1)} + K^{\alpha}_{m}\pi_{\alpha/m})\tau^{\mu\sigma}]\Big\}\Bigg) \\
& \times \exp\left\{\mathrm{i}\int \mathrm{d}^4x[\mathcal{L}^{P}_{eff} + J^{m-1}_{\alpha}\phi^{\alpha}_{(m-1)} + K^{\alpha}_{m}\pi_{\alpha/m}]\right\} \\
= \Bigg(1 + \mathrm{i}\varepsilon_{\sigma}\int \mathrm{d}^4x\Bigg\{ & J^{m-1}_{\alpha}\left(\xi^{\alpha\sigma}_{(m-1)} - \tau^{\mu\sigma}\partial_{\mu}\frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}}\right) \\
& + K^{\alpha}_{m}\left(\eta^{\sigma}_{\alpha/m} - \tau^{\mu\sigma}\partial_{\mu}\frac{\delta}{\mathrm{i}\delta K^{\alpha}_{m}}\right) \\
& + \partial_{\mu}\left[\tau^{\mu\sigma}\left(J^{m-1}_{\alpha}\frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}} + K^{\alpha}_{m}\frac{\delta}{\mathrm{i}\delta K^{\alpha}_{m}}\right)\right]\Bigg\}\Bigg)_{\substack{\phi^{\alpha}_{(m-1)}\to\frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}} \\ \pi_{\alpha/m}\to\frac{\delta}{\mathrm{i}\delta K^{\alpha}_{m}}}} Z[J, K].
\end{aligned} \tag{18}
$$

Consequently, we obtain the following results. If the effective canonical action is invariant under transformation (17) and the Jacobian of this transformation is equal to unity, then the phase-space generating functional of the Green function satisfies the following identities:

$$
\begin{aligned}
\int \mathrm{d}^4x\,\Bigg\{ & J^{m-1}_{\alpha}\left(\xi^{\alpha\sigma}_{(m-1)} - \tau^{\mu\sigma}\partial_{\mu}\frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}}\right) + K^{\alpha}_{m}\left(\eta^{\sigma}_{\alpha/m} - \tau^{\mu\sigma}\partial_{\mu}\frac{\delta}{\mathrm{i}\delta K^{\alpha}_{m}}\right) \\
& + \partial_{\mu}\left[\tau^{\mu\sigma}\left(J^{m-1}_{\alpha}\frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}} + K^{\alpha}_{m}\frac{\delta}{\mathrm{i}\delta K^{\alpha}_{m}}\right)\right]\Bigg\}_{\substack{\phi^{\alpha}_{(m-1)}\to\frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}} \\ \pi_{\alpha/m}\to\frac{\delta}{\mathrm{i}\delta K^{\alpha}_{m}}}} \times Z[J, K] = 0.
\end{aligned} \tag{19}
$$

Expression (19) is called the CWI for global symmetry transformation in extended phase space.

For the internal symmetry transformation $\tau^{\mu\sigma} = 0$, in this case, the identities (19) can be written as

$$
\int \mathrm{d}^4x\,\left\{J^{m-1}_{\alpha}\xi^{\alpha\sigma}_{(m-1)}\left(x, \frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}}, \frac{\delta}{\mathrm{i}\delta K^{\alpha}_{m}}\right) + K^{\alpha}_{m}\eta^{\sigma}_{\alpha/m}\left(x, \frac{\delta}{\mathrm{i}\delta J^{m-1}_{\alpha}}, \frac{\delta}{\mathrm{i}\delta K^{\alpha}}\right)\right\} \times Z[J, K] = 0. \tag{20}
$$

Functionally differentiating (19) with respect to the exterior sources J^0_α many times and setting all exterior sources equal to zero, one can obtain some relationships among the Green functions.

3. Quantal conserved laws

The connection between continuous global symmetries and conservation laws are usually referred to as the first Noether theorem in classical theory. The generalized Noether theorem in canonical formalism for singular higher-order Lagrangian has been derived in previous work [16]. Here the realization of a canonical global symmetry at the quantum level is studied. It is supposed that the effective canonical action is invariant under the transformation (17).

Let us consider the corresponding local transformation in extended phase space:

$$
\begin{aligned}
& x^{\mu'} = x^{\mu} + \Delta x^{\mu} = x^{\mu} + \varepsilon_{\sigma}(x)\tau^{\mu\sigma}(x, \phi^{\alpha}_{(m-1)}, \pi_{\alpha/m}) \\
& \phi^{\alpha'}_{(m-1)}(x') = \phi^{\alpha}_{(m-1)}(x) + \Delta\phi^{\alpha}_{(m-1)}(x) = \phi^{\alpha}_{(m-1)}(x) + \varepsilon_{\sigma}(x)\xi^{\alpha\sigma}_{(m-1)}(x, \phi^{\alpha}_{(m-1)}, \pi_{\alpha/m}) \\
& \pi'_{\alpha/m}(x') = \pi_{\alpha/m}(x) + \Delta\pi_{\alpha/m}(x) = \pi_{\alpha/m}(x) + \varepsilon_{\sigma}(x)\eta^{\sigma}_{\alpha/m}(x, \phi^{\alpha}_{(m-1)}, \pi_{\alpha/m})
\end{aligned} \tag{21}
$$

where $\varepsilon_\sigma(x)(\sigma = 1, 2, \ldots, r)$ are infinitesimal arbitrary functions, whose values and their derivatives up to required order will vanish on the boundary of the time–space domain. Under transformation (21) the variation of the effective canonical action is given by

$$\begin{aligned}\delta I^P_{eff} = \int \mathrm{d}^4 x \varepsilon_\sigma(x) \Bigg\{ & \frac{\delta I^P_{eff}}{\delta\phi^\alpha_{(m-1)}}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma}) + \frac{\delta I^P_{eff}}{\delta\pi_{\alpha/m}}(\eta^\sigma_{\alpha/m} - \pi_{\alpha/m,\mu}\tau^{\mu\sigma}) \\ & + \partial_\mu[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}] + D[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma})]\Bigg\} \\ & + \int \mathrm{d}^4 x\{[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}]\partial_\mu\varepsilon_\sigma(x) \\ & + \pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma}) D\varepsilon_\sigma(x)\}. \end{aligned} \tag{22}$$

Because the effective canonical action is invariant under the global transformation (17), thus the first integral in expression (22) is equal to zero. According to the boundary conditions of $\varepsilon_\sigma(x)$ expression (22) can be written as

$$\begin{aligned}\delta I^P_{eff} &= \int \mathrm{d}^4 x\{[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}]\partial_\mu\varepsilon_\sigma(x) + \pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma}) D\varepsilon_\sigma(x)\} \\ &= -\int \mathrm{d}^4 x \varepsilon_\sigma(x)\{\partial_\mu[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}] \\ &\quad + D[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma})]\}. \end{aligned} \tag{23}$$

The Jacobian of the transformation (21) is denoted by $\bar{J}[\phi, \pi, \varepsilon]$. Substituting (21) and (23) into (8), according to the invariance of the phase-space generating functional (8) under transformation (21), we have

$$\begin{aligned}\int \mathcal{D}\phi^\alpha_{(m-1)}\mathcal{D}\pi_{\alpha/m}\{ & \partial_\mu[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}] \\ & + D[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma})] - J^\sigma_0 - M^\sigma\} \\ & \times \exp\left\{\mathrm{i}\int \mathrm{d}^4 x[\mathcal{L}^P_{eff} + J^{m-1}_\alpha\phi^\alpha_{(m-1)} + K^\alpha_m\pi_{\alpha/m}]\right\} = 0 \end{aligned} \tag{24}$$

where

$$J^\sigma_0 = -\mathrm{i}\delta\bar{J}[\phi, \pi, \varepsilon]/\delta\varepsilon_\sigma(x)|_{\varepsilon_\sigma=0} \tag{25}$$

$$M^\sigma = J^{m-1}_\alpha(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma}) + K^\alpha_m(\eta^\sigma_{\alpha/m} - \pi_{\alpha/m,\mu}\tau^{\mu\sigma}). \tag{26}$$

Functionally differentiating (24) with respect to $J^0_\sigma(x) n$ times, one obtains

$$\begin{aligned}\int \mathcal{D}\phi^\alpha_{(m-1)}\mathcal{D}\pi_{\alpha/m}\Big(& \{\partial_\mu[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}] \\ & + D[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma})] - J^\sigma_0 - M^\sigma\}\phi^\alpha(x_1)\ldots\phi^\alpha(x_n) \\ & - \mathrm{i}\sum_j \phi^\alpha(x_1)\ldots\phi^\alpha(x_{j-1})\phi^\alpha(x_{j+1})\ldots\phi^\alpha(x_n) N^{\alpha\sigma}\delta(x - x_j)\Big) \\ & \times \exp\left\{\mathrm{i}\int \mathrm{d}^4 x[\mathcal{L}^P_{eff} + J^{m-1}_\alpha\phi^\alpha_{(m-1)} + K^\alpha_m\pi_{\alpha/m}]\right\} = 0 \end{aligned} \tag{27}$$

where

$$N^{\alpha\sigma} = \xi^{\alpha\sigma}_0 - \phi^\alpha_{,\mu}\tau^{\mu\sigma}. \tag{28}$$

Setting all exterior sources equal to zero in (27), $J_\alpha^{m-1} = K_m^\alpha = 0$, we obtain

$$\begin{aligned}\langle 0|T^*\{\partial_\mu[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}] + D[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma})] - J_0^\sigma\} \\ \times\phi^\alpha(x_1)\ldots\phi^\alpha(x_n)|0\rangle \\ = \mathrm{i}\sum_j\langle 0|T^*[\phi^\alpha(x_1)\ldots\phi^\alpha(x_{j-1})\phi^\alpha(x_{j+1})\ldots\phi^\alpha(x_n)N^{\alpha\sigma}]|0\rangle\delta(x-x_j)\end{aligned} \tag{29}$$

where the symbol $|0\rangle$ indicates the vacuum state of the field, and the symbol T^* stands for the covariantized T product [9]. Fixing t and letting

$$t_1, t_2, \ldots, t_m \to +\infty, \qquad t_{m+1}, t_{m+2}, \ldots, t_n \to -\infty$$

and using the reduction formula [10]. We find that expression (29) becomes

$$\begin{aligned}\langle \mathrm{out}, m|\{\partial_\mu[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}] + D[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma})] - J_0^\sigma\}|n-m, \mathrm{in}\rangle \\ = 0.\end{aligned} \tag{30}$$

Since m and n are arbitrary, this implies that

$$\partial_\mu[(\pi_{\alpha/m}\phi^\alpha_{(m)} - \mathcal{H}_{eff})\tau^{\mu\sigma}] + D[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),\mu}\tau^{\mu\sigma})] = J_0^\sigma. \tag{31}$$

We now take a cylinder in four-dimensional time–space, the axis of which is directed along the t axis and the upper and lower bottoms V_1 and V_2 are two like-space hypersurfaces $t = t_1$ and $t = t_2$, respectively. If we assume that the fields have a configuration which vanishes rapidly at spatial infinity, then taking the integral of expression (31) on this cylinder, we obtain [16]

$$\int \mathrm{d}^3x[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),k}\tau^{k\sigma}) - \mathcal{H}_{eff}\tau^{0\sigma}]\Big|_{t_1}^{t_2} = \int \mathrm{d}^3x\, J_0^\sigma \tag{32}$$

where the summation over indices k from 1 to 3 are taken repeatedly. Consequently, we obtain the following theorem. If the effective canonical action of a system is invariant under the global transformation (17) and the Jacobian of the corresponding transformation (21) is equal to a constant (or independent of $\varepsilon_\sigma(x)$), then there are some conserved charges at the quantum level for such a system:

$$Q^\sigma = \int \mathrm{d}^3x[\pi_{\alpha/m}(\xi^{\alpha\sigma}_{(m-1)} - \phi^\alpha_{(m-1),k}\tau^{k\sigma}) - \mathcal{H}_{eff}\tau^{0\sigma}] \qquad (\sigma = 1, 2, \ldots, r). \tag{33}$$

These results hold true for the anomaly-free theories.

The QCCs (33) correspond to the classical conservation laws deriving from canonical Noether theorem [16]. In general, H_{eff} differs from canonical Hamiltonian H_c which arise from the effect of quantization of the constrained Hamiltonian system, thus, the QCCs (33) are different from the Noether ones. There is nothing to be surprised at in this result, because of the equations of motion in quantum theories for the constrained Hamiltonian system are different from the classical ones. In classical theories of the constrained Hamiltonian system, Dirac conjectured that all the first-class constraints (primary and secondary) are generators of gauge transformation. In turn, this problem is closely related to the equivalence of Dirac's procedure using the extended Hamiltonian H_{E} and Lagrangian descriptions [23, 24]. From time to time there have been objections to Dirac's conjecture. All these objections are based on the straightforward observation that the equations of the motion deriving from H_{E} are not strictly equivalent to the corresponding Lagrangian equations. In certain cases one can recover from the Hamiltonian equations of motion generated by the total Hamiltonian H_{T} to the corresponding Lagrangian equations of motion. A counter-example was given by one of the authors [25] to show that Dirac's conjecture fails for a system with a singular higher-order Lagrangian. In the quantum theories, based on the invariance of generating functional (8) under the translation of canonical variables $\phi^{\alpha}_{(m-1)}$ and $\pi_{\alpha/m}$, one can proceed in the same way as for the singular first-order Lagrangian to obtain the quantal canonical equations of motion [17]:

$$\dot{\phi}^{\alpha}_{(m-1)} = \frac{\delta H_{eff}}{\delta \pi_{\alpha/m}} \qquad \dot{\pi}_{\alpha/m} = -\frac{\delta H_{eff}}{\delta \phi^{\alpha}_{(m-1)}}. \tag{34}$$

These equations differ from classical ones whether Dirac's conjecture holds true or not. Thus the conserved charges at the quantum level are different from classical ones. In quantum theories, the existence of conserved charges (33) means that effective canonical action (not canonical action) is symmetric under the global transformation (17) and the measure of the functional integral is invariant under the corresponding transformation. Thus, the connection between the symmetries and conservation laws in classical theories in general is no longer preserved in quantum theories.

For the case when the phase-space path integral can be simplified by carrying out explicit integration over canonical momenta, and the phase-space generating functional can be represented in the so-called Lagrangian form of a path integral only over the field variables of the expression containing a certain effective Lagrangian in configuration space. The quantal conserved charges can be obtained by analysing the symmetries of this effective Lagrangian in configuration space. The advantage of the above formulation to obtain conserved charges at the quantum level is that we do not need to carry out explicit integration over the canonical momenta in the phase-space path integral. In general, such integration over canonical momenta cannot be done.

4. Quantal symmetry in configuration space for a gauge-invariant system

For a system with a gauge-invariant Lagrangian $\mathcal{L}$ involving higher-order derivatives of the field variables, the effective Lagrangian $\mathcal{L}_{eff}$ in configuration space can be found by using the FP trick through a transformation of the functional integral [21], $\mathcal{L}_{eff} = \mathcal{L} + \mathcal{L}_f + \mathcal{L}_{gh}$, where $\mathcal{L}_f$ is determined by the gauge conditions and $\mathcal{L}_{gh}$ is a ghost term. The configuration-space generating functional of the Green function for this system can be written as

$$Z[J] = \int \mathcal{D}\phi^{\alpha} \exp\left\{\mathrm{i}\int \mathrm{d}^4x(\mathcal{L}_{eff} + J_{\alpha}\phi^{\alpha})\right\} \tag{35}$$

where ϕ^{α} represents all field variables.

Let us suppose that the effective action is invariant under a global infinitesimal transformation in configuration space. Consider the corresponding local transformation

$$\begin{aligned} x^{\mu} &= x^{\mu} + \Delta x^{\mu} = x^{\mu} + \varepsilon_{\sigma}(x)\tau^{\mu\sigma}(x,\ldots,\phi_{,\mu(m)},\ldots) \\ \phi^{\alpha'}(x') &= \phi^{\alpha}(x) + \Delta\phi^{\alpha}(x) = \phi^{\alpha}(x) + \varepsilon_{\sigma}(x)\xi^{\alpha\sigma}(x,\ldots,\phi_{,\mu(m)},\ldots) \end{aligned} \tag{36}$$

where $\varepsilon_{\sigma}(x)(\sigma = 1, 2, \ldots, r)$ are infinitesimal arbitrary functions, whose values and their derivatives up to required order will vanish on the boundary of the time–space domain. Under transformation (36) the variation of effective action is given by [2]

$$\begin{aligned} \Delta I_{eff} = {}& \int \mathrm{d}^4x\varepsilon_{\sigma}(x)\left\{\frac{\delta I_{eff}}{\delta\phi^{\alpha}}(\xi^{\alpha\sigma} - \phi^{\alpha}_{,\mu}\tau^{\mu\sigma}) + \partial_{\mu}\left[\mathcal{L}_{eff}\tau^{\mu\sigma} + \sum_{n=0}^{N-1}\prod_{\alpha}^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{\alpha\sigma} - \phi^{\alpha}_{,\rho}\tau^{\rho\sigma})\right]\right\} \\ &+ \int \mathrm{d}^4x\left\{\left[\mathcal{L}_{eff}\tau^{\mu\sigma} + \sum_{n=0}^{N-1}\prod_{\alpha}^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{\alpha\sigma} - \phi^{\alpha}_{,\rho}\tau^{\rho\sigma})\right]\partial_{\mu}\varepsilon_{\sigma}(x)\right\}. \end{aligned} \tag{37}$$

Because the effective action is invariant under the global transformation, thus the first integral in expression (37) is equal to zero. Accounting for the boundary conditions of $\varepsilon_{\sigma}(x)$, expression

(37) can be written as

$$\Delta I_{eff} = -\int \mathrm{d}^4x\varepsilon_\sigma(x)\partial_\mu\left[\mathcal{L}_{eff}\tau^{\mu\sigma} + \sum_{n=0}^{N-1}\prod_{\alpha}^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{\alpha\sigma} - \phi^\alpha_{,\rho}\tau^{\rho\sigma})\right]. \tag{38}$$

It is supposed that the Jacobian of the transformation (36) is equal to unity. The generating functional (35) is invariant under transformation (36). We have

$$Z[J] = \int \mathcal{D}\phi^\alpha\left\{1 - \mathrm{i}\int \mathrm{d}^4x\varepsilon_\sigma(x)\partial_\mu\left[\mathcal{L}_{eff}\tau^{\mu\sigma} + \sum_{n=0}^{N-1}\prod_{\alpha}^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{\alpha\sigma} - \phi^\alpha_{,\rho}\tau^{\rho\sigma})\right]\right.$$
$$\left. + \mathrm{i}\int \mathrm{d}^4x J_\alpha\varepsilon_\sigma(x)(\xi^{\alpha\sigma} - \phi^\alpha_{,\mu}\tau^{\mu\sigma})\right\} \times \exp\left\{\mathrm{i}I_{eff} + \mathrm{i}\int \mathrm{d}^4x J_\alpha\phi^\alpha\right\}. \tag{39}$$

The invariance of generating functional (35) under transformation (36) implies that

$$\int \mathcal{D}\phi^\alpha\left\{\partial_\mu\left[\mathcal{L}_{eff}\tau^{\mu\sigma} + \sum_{n=0}^{N-1}\prod_{\alpha}^{\mu\nu(n)}\partial_{\nu(n)}(\xi^{\alpha\sigma} - \phi^\alpha_{,\rho}\tau^{\rho\sigma})\right] - J_\alpha(\xi^{\alpha\sigma} - \phi^\alpha_{,\mu}\tau^{\mu\sigma})\right\}$$
$$\times \exp\left\{\mathrm{i}I_{eff} + \mathrm{i}\int \mathrm{d}^4x J_\alpha\phi^\alpha\right\} = 0. \tag{40}$$

Functionally differentiating (40) with respect to exterior source n times, one can proceed as in section 3. We can obtain the conserved charges at the quantum level in configuration space for such a system:

$$Q^\sigma = \int \mathrm{d}^3x\left[\mathcal{L}_{eff}\tau^{0\sigma} + \sum_{n=0}^{N-1}\prod_{\alpha}^{0\nu(n)}\partial_{\nu(n)}(\xi^{\alpha\sigma} - \phi^\alpha_{,\rho}\tau^{\rho\sigma})\right]. \tag{41}$$

Ward identities in the configuration space can also be deduced using a similar method.

5. YM theory with higher-order derivatives

The YM theory with higher-order derivatives Lagrangian is given by [21]

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4\alpha_0^2}D^a_{b\mu}F^b_{\nu\lambda}D^{a\mu}_c F^{c\lambda\nu} \tag{42}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^a_{bc}A^b_\mu A^c_\nu \tag{43}$$

$$D^a_{b\mu} = \delta^a_b\partial_\mu + f^a_{cb}A^c_\mu. \tag{44}$$

In the Coulomb gauge the generating functional of the Green function for this system can be written as [21]

$$Z[J,\xi,\bar{\xi},\eta] = \int \mathcal{D}A^a_\mu\mathcal{D}A^a_{(1)\mu}\mathcal{D}\pi^\mu_{a/1}\mathcal{D}\pi^\mu_{a/2}\mathcal{D}C_a\mathcal{D}\bar{C}_a\mathcal{D}\lambda_{n'}\delta(\Phi^G_{a1})\delta(\Phi^G_{a2})$$
$$\times \exp\left\{\mathrm{i}\int \mathrm{d}^4x[\bar{\mathcal{L}}^P_{eff} + J^\mu_a A^a_\mu + \bar{\xi}^a C_a + \bar{C}^a\xi_a + \eta^{n'}\lambda_{n'}]\right\} \tag{45}$$

where

$$\bar{\mathcal{L}}^P_{eff} = \mathcal{L}^P + \mathcal{L}_m + \mathcal{L}_{gh} \tag{46}$$

$$\mathcal{L}^P = \pi^\mu_{a/1}\dot{A}^a_\mu + \pi^\mu_{a/2}\dot{A}^a_{(1)\mu} - \mathcal{H}_c \tag{47}$$

$$\mathcal{L}_m = \lambda^a_1\Phi^{(1)}_{a1} + \lambda^a_2\Phi^{(2)}_a \tag{48}$$

$$\mathcal{L}_{gh} = \bar{C}_a D^a_{bi}\partial_i C_b \tag{49}$$

and $\mathcal{H}_c$ is a canonical Hamiltonian density, $\pi^{\mu}_{a/1}$ and $\pi^{\mu}_{a/2}$ are canonical momenta conjugated to A^a_μ and $A^a_{(1)\mu} = \dot{A}^a_\mu$, respectively, Φ and Φ^G are constraints and gauge conditions, respectively. In expression (45) we have introduced exterior sources J^μ_a only to field A^a_μ. The theory is independent of the choice of gauge constraints [26], the $\Phi^G_{ai}(i = 1, 2)$ can be replaced by $\Phi^{G'}_{ai} = \Phi^G_{ai} - P_{ai}(x)$, where $P_{ai}(x)$ are independent of the gauge. Multiplying (45) by $\exp[-\frac{1}{2\alpha_i}\int d^4x(P_{ai})^2]$ (α_i are parameters) and taking the path integral with respect to $P_{ai}(x)$, we can obtain

$$Z[J,\xi,\bar{\xi},\eta] = \int \mathcal{D}A^a_\mu \mathcal{D}A^a_{(1)\mu}\mathcal{D}\pi^\mu_{a/1}\mathcal{D}\pi^\mu_{a/2}\mathcal{D}C_a\mathcal{D}\bar{C}_a\mathcal{D}\lambda_{n'} \times \exp\left\{ \mathrm{i}\int d^4x[\mathcal{L}^P_{eff} + J^\mu_a A^a_\mu + \bar{\xi}^a C_a + \bar{C}^a\xi_a + \eta^{n'}\lambda_{n'}]\right\} \tag{50}$$

where

$$\mathcal{L}^P_{eff} = \bar{\mathcal{L}}^P_{eff} + \mathcal{L}_f \tag{51}$$

$$\mathcal{L}_f = -\frac{1}{2\alpha_1}(\Phi^G_{a1})^2 - \frac{1}{2\alpha_2}(\Phi^G_{a2})^2. \tag{52}$$

It is easy to check that $\mathcal{L}^P$ and $\mathcal{L}_{gh}$ are invariant under the following transformation [17, 27]:

$$A^{a'}_\mu(x) = A^a_\mu(x) + D^a_{\sigma\mu}\varepsilon^\sigma(x) \tag{53a}$$

$$A^{a'}_{(1)\mu}(x) = A^a_{(1)\mu}(x) + \partial_0 D^a_{\sigma\mu}\varepsilon^\sigma(x) \tag{53b}$$

$$\pi^{\mu'}_{a/1}(x) = \pi^\mu_{a/1}(x) + f^a_{\sigma c}\pi^\mu_{c/1}(x)\varepsilon^\sigma(x) + f^a_{\sigma c}\mu^\mu_{c/2}(x)\dot{\varepsilon}^\sigma(x) \tag{53c}$$

$$\pi^{\mu'}_{a/2}(x) = \pi^\mu_{a/2}(x) + f^a_{\sigma c}\pi^\mu_{c/2}(x)\varepsilon^\sigma(x) \tag{53d}$$

$$C^{a'}(x) = C^a(x) + \mathrm{i}g(T_\sigma)_{ab}C^b(x)\varepsilon^\sigma(x) \tag{53e}$$

$$\bar{C}^{a'}(x) = \bar{C}^a(x) - \mathrm{i}g\bar{C}^b(x)(T_\sigma)_{ba}\varepsilon^\sigma(x) + \frac{\mathrm{i}g}{\Box}\partial_\mu[\bar{C}^b(x)(T_\sigma)_{ba}\partial^\mu\varepsilon^\sigma(x)] \tag{53f}$$

where T_σ are representation matrices of the generators of gauge group. The expression (53*f*) can be written as

$$\bar{C}^{a'} = \bar{C}^a(x) - \mathrm{i}g\bar{C}^b(x)(T_\sigma)_{ba}\varepsilon^\sigma(x) + \mathrm{i}g\int d^4y\Delta_0(x,y)\partial_\mu[\bar{C}^b(x)(T_\sigma)_{ba}\partial^\mu\varepsilon^\sigma(x)] \tag{54}$$

where

$$\Box\Delta_0(x,y) = \mathrm{i}\delta^{(4)}(x,y). \tag{55}$$

The change of $\mathcal{L}_m + \mathcal{L}_f$ is denoted by

$$\delta(\mathcal{L}_m + \mathcal{L}_f) = F_\sigma(\lambda, A, \dot{A}, \pi_{a/1}, \pi_{a/2})\varepsilon^\sigma(x) \tag{56}$$

under transformation (53), where F_σ depends on multiplier fields, λ_n, and canonical variables. Since $J^0_\sigma = 0$ for the transformation (53) [17, 28], according to the invariant of the generating functional (50) under the transformation (53), we can obtain the CWI for the transformation (53),

$$\left\{\mathrm{i}F_\sigma - \mathrm{i}\partial_\mu J^\mu_\sigma + gf^a_{\sigma c}J^\mu_a\frac{\delta}{\delta J^\mu_c} + \mathrm{i}g\bar{\xi}_a(T_\sigma)_{ab}\frac{\delta}{\delta\bar{\xi}_b} - \mathrm{i}g\xi_a(T_\sigma)_{ba}\frac{\delta}{\delta\xi_b} + \mathrm{i}g\partial_\mu\left[\partial^\mu\left(\xi_a\frac{1}{\Box}\right)(T_\sigma)_{ba}\frac{\delta}{\delta\xi_b}\right]\right\} Z[J,\xi,\bar{\xi},\eta] = 0. \tag{57}$$

As usual we let $Z[J,\xi,\bar{\xi},\eta] = \exp\{\mathrm{i}W[J,\xi,\bar{\xi},\eta]\}$ and use the definition of generating functional of proper vertices $\Gamma[A,\bar{C},C,\lambda]$ which is given by performing a functional Legendre transformation of $W[J,\xi,\bar{\xi},\eta]$. Then, the CWI (57) can be written as

$$\mathrm{i}F_\sigma + \mathrm{i}\partial_\mu \frac{\delta\Gamma}{\delta A^\sigma_\mu} - \mathrm{i}g f^a_{\sigma c} A^c_\mu \frac{\delta\Gamma}{\delta A^a_\mu} - \mathrm{i}g C^a (T_\sigma)_{ab} \frac{\delta\Gamma}{\delta C^b} + \mathrm{i}g \bar{C}^a (T_\sigma)_{ba} \frac{\delta\Gamma}{\delta \bar{C}^b}$$
$$-\mathrm{i}g\partial^\mu \left[\partial_\mu \left(\frac{\delta\Gamma}{\delta\bar{C}^a}\frac{1}{\Box}\right)(T_\sigma)_{ba}\bar{C}^b\right] = 0. \tag{58}$$

We functionally differentiate (58) with respect to $\bar{C}^k(x_2)$ and $\bar{C}^m(x_3)$, and set all fields equal to zero, $A = C = \bar{C} = \lambda = 0$. Then, we obtain

$$\partial^\mu_{x_1} \frac{\delta^3\Gamma[0]}{\delta\bar{C}^k(x_2)\delta C^m(x_3)\delta A^\mu_\sigma(x_1)} - g(T_\sigma)_{mb} \frac{\delta^2\Gamma[0]}{\delta\bar{C}^k(x_2)\delta C^b(x_1)}\delta(x_1 - x_3)$$
$$+g(T_\sigma)_{bk} \frac{\delta^2\Gamma[0]}{\delta\bar{C}^b(x_1)\delta C^m(x_3)}\delta(x_1 - x_2)$$
$$-g\partial^\mu \left[\partial_\mu \left(\frac{\delta^2\Gamma[0]}{\delta\bar{C}^a(x_1)\delta C^m(x_2)}\frac{1}{\Box}\right)(T_\sigma)_{ka}\delta(x_1 - x_2)\right] = 0. \tag{59}$$

Functionally differentiating (59) many times with respect to the field variables, one can obtain various Ward identities for proper vertices.

Expression (59) is a new form of the Ward identities for gauge-ghost proper vertices which differs from the Ward–Takahashi identities arising from the BRS invariance for an effective Lagrangian in configuration space. The BRS transformation is nonlinear in ghost fields, while the transformation (53) is a linear (non-local) one. The above formulation to derive the Ward identities for proper vertices has significant advantage in that one does not carry out the integration over canonical momenta in phase-space path integral. In a general case this integration cannot be carried out. The invariance of the terms $\mathcal{L}^P$ and $\mathcal{L}_{gh}$ in (46) under the transformation (53) is only required for deriving (59). This is also different from BRS invariance for an effective Lagrangian. A similar problem in QCD has been discussed by Kuang and Yi using the configuration space generating functional [28].

Let us put $\varepsilon^\sigma(x) = \varepsilon^\nu_0 A^\sigma_\nu(x)$, where ε^ν_0 are parameters, then, the transformation (53) will be converted into a global one. In this case, from the CWI (20) for global transformation we have

$$\int \mathrm{d}^4x \left\{F_\nu - \partial_\mu J^\mu_\sigma \frac{\delta}{\delta J^\nu_\sigma} - \mathrm{i}g f^a_{\sigma c} J^\mu_a \frac{\delta}{\delta J^\mu_c}\frac{\delta}{\delta J^\nu_\sigma} + g\bar{\xi}_a (T_\sigma)_{ab} \frac{\delta}{\delta\bar{\xi}_b}\frac{\delta}{\delta J^\nu_\sigma} - g\bar{\xi}_a (T_\sigma)_{ab}\frac{\delta}{\delta\xi_b}\frac{\delta}{\delta J^\nu_\sigma}\right.$$
$$\left. + \frac{g}{\Box}\partial_\mu \left[\frac{\delta}{\delta\xi_b}(T_\sigma)_{ba}\xi_a \partial^\mu \frac{\delta}{\delta J^\nu_\sigma}\right]\right\} Z[J,\xi,\bar{\xi},\eta] = 0 \tag{60}$$

where F_ν satisfy the relation $\delta(\mathcal{L}_m + \mathcal{L}_f) = \varepsilon^\nu_0 F_\nu(\lambda, A, \dot{A}, \pi_{a/1}, \pi_{a/2})$. The expression (60) can be written in terms of $\Gamma[A,\bar{C},C,\lambda]$ as

$$\int \mathrm{d}^4x \left\{F_\nu - A^\sigma_\nu \partial_\mu \frac{\delta\Gamma}{\delta A^\sigma_\mu} - \mathrm{i}g f^a_{\sigma c} A^\sigma_\nu A^c_\mu \frac{\delta\Gamma}{\delta A^a_\mu} + gA^\sigma_\nu C^a (T_\sigma)_{ab}\frac{\delta\Gamma}{\delta C^b} - gA^\sigma_\nu \bar{C}^a (T_\sigma)_{ba}\frac{\delta\Gamma}{\delta\bar{C}^b}\right.$$
$$\left. + gA^\sigma_\nu \partial_\mu \left[\partial_\mu \left(\frac{\delta\Gamma}{\delta\bar{C}^a}\frac{1}{\Box}\right)(T_\sigma)_{ba}\bar{C}^b\right]\right\} Z[J,\xi,\bar{\xi},\eta] = 0. \tag{61}$$

Functionally differentiating (61) with respect to $A_\lambda(x_1)$, $\bar{C}^k(x_2)$ and $C_m(x_3)$ and setting all fields equal to zero, we can also obtain Ward identities (59) for gauge-ghost proper vertices. That is to say, the identities (59) can be derived from both the CWIs for local and global transformation.

Furthermore, $\mathcal{L}^P$ and $\mathcal{L}_{gh}$ are also invariant under this global transformation. The variations of first-class constraints under the gauge transformation (53*a*)–(53*d*) are within the constraint hypersurface [17], thus $\delta\mathcal{L}_m \approx 0$ and $\delta\mathcal{L}_f \approx 0$ under transformation (53), i.e. $\delta I_{eff} \approx 0$ under the transformation (53). From (33) and (53) we obtain the QCC for YM field with higher-order derivatives

$$Q_\nu = \int \mathrm{d}^4x \Bigg\{ \pi^\mu_{a/1} D^a_{\sigma\mu} A^\sigma_\nu + \pi^\mu_{a/2} \partial_0 (D^a_{\sigma\mu} A^\sigma_\nu) + \mathrm{i}g\pi_a (T_\sigma)_{ab} C^b A^\sigma_\nu \\ -\mathrm{i}g\bar{\pi}_a \left[\bar{C}^b (T_\sigma)_{ba} A^\sigma_\nu - \int \mathrm{d}^4y \Delta_0(x,y) \partial_\mu (\bar{C}^b(x) (T_\sigma)_{ba} \partial^\mu A^\sigma_\nu(x) \right] \Bigg\} \tag{62}$$

where $\pi^\mu_{a/1}$, $\pi^\mu_{a/2}$, π_a and $\bar{\pi}_a$ are canonical momenta conjugate to fields A^μ_a, $\dot{A}^\mu_a$, C^a and $\bar{C}^a$, respectively.

$$\pi^0_{a/1} = \frac{1}{\alpha_0^2} D^b_{aj} D^c_{b0} F^{jo}_c \tag{63a}$$

$$\pi^i_{a/1} = \frac{1}{\alpha_0^2} (D^{bj}_a D^e_{bj} F^{oi}_e + D^a_{bj} D^e_{bo} F^{ij}_e) - D^b_{a0} \pi^i_{b/2} + F^{oi}_a \tag{63b}$$

$$\pi^0_{a/2} = 0 \tag{63c}$$

$$\pi^i_{a/2} = \frac{1}{\alpha_0^2} D^a_{bj} F^{ij}_b \tag{63d}$$

$$\pi_a = -\dot{\bar{C}}^a \tag{63e}$$

$$\bar{\pi}_a = D^a_{b0} C^b. \tag{63f}$$

6. Non-Abelian higher-derivative CS theory

Non-Abelian CS gauge fields coupled to the Fermi field have been studied [29]. The quantal canonical symmetries for such a system have been discussed in previous work [30]. Now, we further study symmetry of the CS gauge field A^a_μ coupled to the scalar field ϕ whose Lagrangian is given by

$$L = (D_\mu\phi)^+ (D^\mu\phi) - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{\kappa}{4\pi} \varepsilon^{\mu\nu\rho} \left(\partial_\mu A^a_\nu A^a_\rho + \frac{1}{3} f^{abc} A^a_\mu A^b_\nu A^c_\rho \right) \\ - \frac{c^2}{4\pi} D_\rho F^a_{\mu\nu} D^\rho F^{a\mu\nu} \tag{64}$$

where

$$D_\mu \phi^\alpha = \partial_\mu \phi^\alpha + g A^\gamma_\mu T^\gamma_{\alpha\beta} \phi^\beta$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$$

and T^a is a generator of gauge group. The gauge invariance of the non-Abelian CS term requires the quantization of the dimensionless constant κ, $\kappa = \frac{n}{4\pi} (n \in Z)$ [31].

According to the Ostrogradsky transformation one can introduce the canonical momenta $P^{a\mu}$, $Q^{a\mu}$, π, π^+ with respect to A^a_μ, $B^a_\mu = \dot{A}^a_\mu$, ϕ, ϕ^+ respectively, and

$$P^{a\mu} = F^{a\mu 0} + \frac{\kappa}{4\pi} \varepsilon^{0\mu\nu} A^a_\nu - \frac{c^2}{\pi} D_i D^i F^{a\mu 0} - D_0 Q^{a\mu} - \frac{c^2}{\pi} D_i D_0 F^{a\mu i} + f^{abc} A^c_0 Q^{b\mu}$$

$$Q^{a\mu} = \frac{c^2}{\pi} D_0 F^{a\mu 0}$$

$$\pi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = (D_0\phi)^+$$

$$\pi^+ = \frac{\partial\mathcal{L}}{\partial\dot{\phi}^+} = D_0\phi.$$

The constraints in phase space are:

$$\Lambda^{(0)a} = Q^{a0} \approx 0 \tag{65}$$

$$\Lambda^{(1)a} = -P^{a0} + D_i Q^{ai} \approx 0 \tag{66}$$

$$\Lambda^{(2)a} = -D_i P^{ai} - \frac{\kappa}{4\pi}\partial_i A_j^a \varepsilon^{ij} - f^{abc} B_i^b Q^{ci} - f^{abc} A_0^b D_i Q^{ci} \approx 0. \tag{67}$$

It is easy to check that $\Lambda^{(0)}$, $\Lambda^{(1)a}$, $\Lambda^{(2)a}$ are first-class constraints. A corresponding gauge condition can be chosen that

$$\Omega_0^a = B_0^a \approx 0 \tag{68a}$$

$$\Omega_1^a = \partial_i B^{ai} \approx 0 \tag{68b}$$

$$\Omega_2^a = \partial_i A^{ai} \approx 0. \tag{68c}$$

Using the Faddeev–Senjavonic method [29], the phase-space generating functional for this model can be written as

$$Z[J] = \int \mathcal{D}A_\mu^a \mathcal{D}P^{a\mu}\mathcal{D}B_\mu^a\mathcal{D}Q^{a\mu}\mathcal{D}\phi\mathcal{D}\pi\mathcal{D}\phi^+\mathcal{D}\pi^+\delta(\Lambda_l)\delta(\Omega_l)\det\{\Lambda,\Omega\}$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}^3x(B_\mu^a P^{a\mu} + \dot{B}_\mu^a Q^{a\mu} + \pi\dot{\phi} + \pi^+\dot{\phi}^+\right.$$
$$\left. -\mathcal{H}_c + J_{1a}^\mu A_\mu^a + J_{2a}^\mu B_\mu^a + J_1\phi + J_1^+\phi^+)\right\} \tag{69}$$

where $\det\{\Lambda,\Omega\} = \det A\cdot\det M^{ab}\cdot\det M^{ab}$, $A = -\delta^{ab}\delta(x-y)$. Let $M_c = (M^{ab})$; factor $\det M_c\delta(\partial_i A^{ai})$ can be replaced by $\det M_l\delta(\partial_\mu A^{a\mu})$ [29], and

$$M_l = (\delta^{ab}\partial_\mu\partial^\mu - f^{abc}A_\mu^c\partial^\mu)\delta(x-y).$$

Using the integral properties of Grassmann variables $C_l(x)$ and $\bar{C}_k(x)$, one gets

$$Z[J] = \int \mathcal{D}A_\mu^a\mathcal{D}P^{a\mu}\mathcal{D}B_\mu^a\mathcal{D}Q^{a\mu}\mathcal{D}\phi\mathcal{D}\pi\mathcal{D}\phi^+\mathcal{D}\pi^+\mathcal{D}\lambda\mathcal{D}C^a\mathcal{D}\bar{C}^a$$
$$\times\exp\left\{\mathrm{i}\int\mathrm{d}^3x[\mathcal{L}_{eff}^P + J_{1a}^\mu A_\mu^a + J_{2a}^\mu B_\mu^a + J_1\phi + J_1^+\phi^+ + \bar{J}_{3a}C^a + \bar{C}^a J_{3a}]\right\} \tag{70}$$

where

$$\begin{aligned}
\mathcal{L}_{eff}^P &= \mathcal{L}^P + \mathcal{L}_g + \mathcal{L}_{gh} + \mathcal{L}_m\\
\mathcal{L}^P &= B_\mu^a P^{a\mu} + \dot{B}_\mu^a Q^{a\mu} + \dot{\phi}\pi + \dot{\phi}^+\pi^+ - \mathcal{H}_c\\
\mathcal{L}_g &= -\frac{1}{2\alpha_2}(\Omega_2^a)^2 = -\frac{1}{2\alpha_2}(\partial_\mu A^{a\mu})^2\\
\mathcal{L}_{gh} &= -\partial^\mu\bar{C}^a D_{b\mu}^a C_b\\
\mathcal{L}_m &= \lambda_0^a\Lambda^{(0)a} + \lambda_1^a\Lambda^{(1)a} + \lambda_2^a\Lambda^{(2)a} - \frac{1}{2\alpha_0}(\Omega_0^a)^2 - \frac{1}{2\alpha_1}(\Omega_1^a)^2.
\end{aligned} \tag{71}$$

Under the transformation in the phase space (where τ is Grassmann's parameter)

$$\begin{aligned}
&\delta\phi = -\mathrm{i}\tau T^a C^a\phi && \delta\pi = \mathrm{i}\tau T^a C^a\pi\\
&\delta\phi^+ = \mathrm{i}\tau T^a C^a\phi^+ && \delta\pi^+ = -\mathrm{i}\tau T^a C^a\pi^+\\
&\delta A_\mu^a = -\tau D_{b\mu}^a C^b && \delta P^{a\mu} = f_{be}^a P^{e\mu}C^b\tau - f_{be}^a Q^{e\mu}\dot{C}^b\\
&\delta B_\mu^a = \partial_0(-\tau D_{b\mu}^a C^b) && \delta Q^{a\mu} = f_{be}^a Q^{e\mu}C^b\tau\\
&\delta C^a = \tfrac{1}{2}f^{a\beta\gamma}C_\beta C_\gamma && \delta\bar{C}^a = -\frac{1}{\alpha_2}\partial^\mu A_\mu^a.
\end{aligned} \tag{72}$$

The effective canonical Lagrangian $\mathcal{L}_{eff}^{P}$ is invariant on the constraint hypersurface determined by the constraints and the Jacobian of transformation (72) is equal to unity. From expression (33), one obtains the BRS conserved quantity for non-Abelian higher-derivative CS theory at the quantum level:

$$Q = \int \mathrm{d}^2x(P^{a\mu}\delta A_\mu^a + Q^{a\mu}\delta B_\mu^a + \pi\delta\phi + \delta\phi^+\pi^+ + \bar{R}_a\delta C^a + \delta\bar{C}^a R_a) \tag{73}$$

where $\bar{R}_a$ and R_a are the canonical momenta conjugate to C^a and $\bar{C}^a$, respectively.

The effective canonical Lagrangian is also invariant under the spatial rotation transformation in the (x_1, x_2) plane, and the Jacobian of the transformation of the fields is equal to unity, and $\tau^{0\sigma} = 0$ in the spatial rotation. Thus, from expression (33) we obtain the conserved angular momentum for non-Abelian higher-derivative CS theory at the quantum level

$$\begin{aligned} J_{12} = \int \mathrm{d}^2x \Bigg\{ & P^{a\mu}\left(x_2\frac{\partial A_\mu^a}{\partial x_1} - x_1\frac{\partial A_\mu^a}{\partial x_2}\right) + Q^{a\mu}\left(x_2\frac{\partial B_\mu^a}{\partial x_1} - x_1\frac{\partial B_\mu^a}{\partial x_2}\right) + P^{a\mu}\left(\sum_{12}\right)_{\mu\nu} A_\nu^a \\ & + Q^{a\mu}\left(\sum_{12}\right)_{\mu\nu} B_\nu^a + \pi\left(x_2\frac{\partial\phi}{\partial x_1} - x_1\frac{\partial\phi}{\partial x_2}\right) + \left(x_2\frac{\partial\phi^+}{\partial x_1} - x_1\frac{\partial\phi^+}{\partial x_2}\right)\pi^+ \\ & + \bar{R}_a\left(x_2\frac{\partial C^a}{\partial x_1} - x_1\frac{\partial C^a}{\partial x_2}\right) + \left(x_2\frac{\partial\bar{C}^a}{\partial x_1} - x_1\frac{\partial\bar{C}^a}{\partial x_2}\right) R_a \Bigg\} \end{aligned} \tag{74}$$

where $(\sum_{jk})_{\mu\nu} = g_{j\mu}g_{k\nu} - g_{j\nu}g_{k\mu}$.

Using the formulation discussed in section 4, we can obtain the same results from the configuration-space generating functional. This implies that the FP trick is valid for these higher-order derivative non-Abelian CS theories. We also see that the quantal conserved angular momentum differs from classical Noether one [2] in that one needs to take into account the contribution of angular momentum of ghost fields in non-Abelian higher-derivative CS theory. For the case of the first-order derivative non-Abelian CS theories, $c = 0$, one can proceed in the same way to obtain the results. This problem has been discussed in classical theories [32, 33]. We do not think the conclusions in classical theories are always valid at the quantum level. The properties of fractional spin and statistics in non-Abelian CS theory needs further study.

The advantage of the canonical formalism to derive the conservation laws at the quantum level is that one does not need to carry out the integration over the canonical momenta in the phase-space path integral.

It had been pointed out that the anomalies can be viewed as being a result of the non-invariance of the functional measure under some symmetry transformation [34]. The above results indicate that the anomalies may be appearing in a case with an invariance of the functional measure under the symmetry transformation.

7. Conclusions

In the theory of path integral quantization for a dynamical system, the phase-space path integrals are more basic than configuration-space path integrals. Based on the phase-space generating functional of the Green function for a system with a singular higher-order Lagrangian, the canonical symmetry for such a system at the quantum level is studied. The CWIs for the local and global transformation in extended phase space have been deduced, respectively. The QCC under the global symmetry transformation in extended phase space has been also deduced. The existence of this conserved quantity means that the effective canonical action is symmetric and constraint conditions are preserved in the constrained hypersurface under the transformation, and that the Jacobian of the corresponding transformation is equal to unity. In general, this conserved charge at the quantum level differs from the Noether ones in classical theories. The connection between symmetry and conservation law in classical theories in general is no longer preserved in quantum theories. The advantage of our canonical formalism is that one does not need to carry out the integration over canonical momenta in the phase-space path integral as usual. In general, that integration cannot be carried out. The application of the above results to YM and CS theory with higher-order derivatives have been given, a new form of gauge-ghost proper vertices was derived from both the CWI for local and global transformation, and new conserved charges at quantum level were obtained in YM theory. The quantal BRS conserved charge and conserved angular momentum were also derived in CS theory: this conserved angular momentum differs from the classical Noether one. For the gauge-invariant system, a comparison of the quantal conservation laws in canonical formalism with the results deriving from the FP trick in configuration-space path integral were discussed.

Numerous recent investigation of (2 + 1)-dimensional gauge theories with CS terms in the Lagrangian have revealed the occurrence of fractional spin and statistics [35, 36]. In those papers the angular momenta were deduced by using the classical Noether theorem. We have shown that the conclusion holds true for Abelian CS theory at the quantum level [37]. But whether the results are valid or not at the quantum level for non-Abelian CS theory requires further study. Work along these lines is in progress.

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References

[1] Noether E 1981 *Nach. Akad. Wiss. Gott. Math. Phys.* Kl. II **1918** 235
[2] Li Z-P 1993 *Classical and Quantal Dynamics of Constrained Systems and Their Symmetry Properties* (Beijing: Beijing Polytechnic University Press)
[3] Ward J C 1950 *Phys. Rev.* **77** 2931
[4] Takahashi Y 1957 *Nuovo Cimento* **6** 371
[5] Slavnov A A 1972 *Theor. Math. Phys.* **10** 99
[6] Taylor J C 1971 *Nucl. Phys.* B **33** 436
[7] Joglekar S D 1991 *Phys. Rev.* D **44** 3879
[8] Danilov G S 1991 *Phys. Lett.* B **257** 285
[9] Suura H and Yang B-L 1973 *Phys. Rev.* D **8** 4353
[10] Yong B-L 1987 *Introduction to Quantum Field Theories* (Beijing: Science Press)
[11] Lhallabi T 1989 *Int. J. Theor. Phys.* **28** 875
[12] Mizrahi M M 1978 *J. Math. Phys.* **19** 298
[13] Lee T D and Yang C N 1962 *Phys. Rev.* **128** 885
[14] Gerstein I S, Jackiw R, Lee B W and Weinberg S 1971 *Phys. Rev.* D **3** 2486
[15] Du T-S, Yin H-C and Ruan T-N 1980 *Nucl. Phys.* B **164** 103
[16] Li Z-P 1993 *Sci. China* A **36** 1212
Li Z-P 1994 *Phys. Rev.* E **50** 876
[17] Li Z-P 1995 *Int. J. Theor. Phys.* **34** 523
[18] Li Z-P 1994 *Europhys. Lett.* **27** 563
[19] Li Z-P 1997 *Z. Phys.* C **76** 181
[20] Saito Y, Sugano R, Ohta T and Kimura T 1989 *J. Math. Phys.* **30** 1122
[21] Gitman D M and Tyutin I V 1990 *Quantization of Fields with Constraints* (Berlin: Springer)
[22] Li Z-P 1987 *Int. J. Theor. Phys.* **26** 853
[23] Costa M E V, Girotti H O and Simoes T J M 1985 *Phys. Rev.* D **32** 405

[24] Henneaux M, Teitelboim C and Zanell J 1990 *Nucl. Phys.* B **332** 169
[25] Li Z-P 1993 *Europhys. Lett.* **21** 141
[26] Sundermeyer K 1982 *(Lecture Notes in Physics)* (Berlin: Springer) p 169
[27] Li Z-P and Xie Y C 1994 *Commun. Theor. Phys.* **21** 247
[28] Kungang Y-P and Yi Y-P 1980 *Phys. Energ. Fortis Phys. Nucl.* **4** 286
[29] Foussats A *et al* 1995 *Int. J. Theor. Phys.* **34** 1037
Foussats A *et al* 1996 *J. Math. Phys.* **37** 84
[30] Li Z-P and Bao J 1997 *Europhys. Lett.* **39** 599
[31] Deser S, Jackiw R and Templeton S 1982 *Ann. Phys., NY* **140** 372
[32] Antillon A *et al* 1995 *Phys. Lett.* B **359** 327
[33] Banerjee R and Chakraborty B 1996 *Ann. Phys.* **247** 188
[34] Fujikawa K 1980 *Phys. Rev. Lett.* **44** 1733
Fujikawa K 1981 *Phys. Rev.* D **23** 2262
[35] Banerjee R 1993 *Phys. Rev.* D **48** 2905
Banerjee R 1994 *Nucl. Phys.* B **419** 611
[36] Kim J K, Kim W-T and Shin H J 1994 *J. Phys. A: Math. Gen.* **27** 6067
[37] Li Z-P 1996 *Sci. China* A **39** 738